Code No.	N – 3579

Entrance Examination for Admission to the P.G. Courses in the Teaching Departments, 2022

CSS

MATHEMATICS/MATHEMATICS WITH FINANCE AND COMPUTATION

General Instructions

1. The Question Paper is having two Parts — Part 'A' Objective type (60%) & Part 'B' Descriptive type (40%).
2. Objective type questions which carry 1 mark each are to be (✓) 'tick marked' in the response sheets against the appropriate answers provided.
3. 8 questions are to be answered out of 12 questions carrying 5 marks each in Part 'B'.
4. **Negative marking** : 0.25 marks will be deducted for each wrong answer in Part 'A'.

Time : 2 Hours **Max. Marks : 100**

To be filled in by the Candidate									
Register Number	in Figures								
	in words								

PART – A

(Objective Type)

Choose appropriate answer from the options in the questions. **One** mark **each**.

(60 $\times$ 1 = 60 marks)

1. For any vector $\bar{a}$; $|i\times\bar{a}|^2+|j\times\bar{a}|^2+|k\times\bar{a}|^2$ is equal to

   a) $|\bar{a}|^2$

   b) $3\bar{a}^2$

   c) $2|\bar{a}|^2$

   d) $\frac{|\bar{a}|^2}{3}$

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2. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then curl $\bar{r}$ is

   a) 3 b) 0

   c) 1 d) 2

3. The directional derivative of $\phi = x^2 + z^2 - y^2$ at the point (1, 3, 2) is the maximum in the direction

   a) $i - 2j + 2k$ b) $2i - 6j + 4k$

   c) $2i + 6j - 4k$ d) $2i - 6j - 4k$

4. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, and $|\bar{r}| = r$ then which of the following is not true

a) $\text{div}\,\bar{r} = 3$ b) $\text{curl}\;\bar{r} = 0$

c) $\text{grad}\; r = \bar{r}$ d) $\text{div curl}\;\bar{r} = 0$

5. Let $\bar{a}$ and $\bar{b}$ be two non zero vectors such that $|\bar{a} + \bar{b}| = |\bar{a} - \bar{b}|$. Then

a) $\bar{a} \perp \bar{b}$ b) $\bar{a} \,||\, \bar{b}$

c) $\bar{a} = \bar{b}$ d) $\bar{a} \cdot \bar{b} = 1$

6. The vector $\dfrac{\bar{r}}{r^3}$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $|\bar{r}| = r$ is

a) only irrotational b) only solenoidal

c) irrotational and solenoidal d) neither irrotational nor solenoidal

7. $x^2 - y^2 + 2y + k = 0$ represents a pair of perpendicular straight lines the k equals

a) 1 b) 4

c) –1 d) 0

8. Find the value of k so that the equation $x^2 - kxy + 2y^2 + 3x - 5y + 2 = 0$ represents a pair of straight lines are

a) $\dfrac{3}{2}, 9$ b) $\dfrac{3}{2}, -3$

c) $\dfrac{9}{2}, 3$ d) $\dfrac{9}{2}, -3$

9. Angle between the lines whose direction cosines l, m, n satisfy $2l + 2m - n = 0, mn + nl + lm = 0$ is

a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{4}$

c) $\dfrac{\pi}{6}$ d) $\dfrac{\pi}{2}$

10. Find the critical numbers of $f(x) = x - 3x^{\frac{1}{3}}$.

a) 1, 0 b) 1, –1

c) 1, 0, –1 d) none of these

11. Determine the intervals in which $f(x) = x^3 - 3x^2 + 2$ is decreasing.

a) $(-\infty, 0)$ b) $(2, \infty)$

c) $(0, 2)$ d) $(0, \infty)$

12. The limit $\lim_{n \to \infty} \left(e + \left(\frac{2}{3} \right)^n \right)^4$ is

a) e b) $1 + e$

c) $(1+e)^4$ d) e^4

13. An object moving in a straight line has velocity $v = 5t^4 + 3t^2$ at time t. How far does the object travel between $t = 1$ and $t = 2$?

a) 38 units b) 36 units

c) 37.5 units d) 38.3 units

14. A parabolic doorway with base 6 feet and height 8 feet is cut out of a wall. How many square feet of wall space are removed?

a) 32 sq.feet b) 33 sq.feet

c) 30 sq.feet d) 31 sq.feet

15. Find the slope of the tangent line to the graph $f(x) = x^2$ at the point (1, 1) is

a) 2 b) –2

c) $\frac{1}{2}$ d) $-\frac{1}{2}$

16. The extreme values of $f(x) = -2\cos x - x$ on $[0, 2\pi]$ is

a) 2 and –5.4 b) 2 and –4.03

c) 0 and –5.4 d) –2 and 5.4

17. Equation of the tangent line to the graph of the equation $y = -x^2 + 4x$ at the point (2, 4) is

a) $y = 3$ b) $x = 3$

c) $y = -4$ d) $y = 4$

18. The function $f(x) = \frac{\sqrt{x}}{\sin x}$ is continuous on

a) $[0, \infty)$ b) $[0, \pi]$

c) $(0, \pi]$ d) $(0, \pi)$

19. The average value of $f(x) = 4 - x^2$ over the interval [–1, 3] is

a) $\frac{5}{3}$ b) $\frac{2}{3}$

c) $\frac{1}{3}$ d) $\frac{2}{5}$

20. What is the order of the differential equation $\frac{d^4y}{dx^2} + 3\left(\frac{d^2y}{dx^2}\right)^5 + 5 = y = 0$

a) 2 b) 4

c) 5 d) 1

21. The differential equation regarding the family of curves $y = e^{5x}$ is

a) $x\frac{dy}{dx} = x\log x$

b) $y\frac{dy}{dx} = y\log y$

c) $x + \frac{dy}{dx} = y\log y$

d) $x\frac{dy}{dx} = y\log y$

22. Integrating factor of the differential equation $x\log x\frac{dy}{dx} + y = 2\log x$

a) $\log x$ b) e^x

c) $\log(\log x)$ d) x

23. General solution of the differential equation $xy\frac{dy}{dx} - 1 = 0$ is

a) $xy = \log x + C$ b) $\frac{x^2}{2} = \log y + C$

c) $\frac{y^2}{2} = \log x + C$ d) none of these

24. Which of the following is an exact differential equation?

a) $xe^{x}dx = e^{y}dy$

b) $(2xy - 3x^2)\, dx + (x^2 - 2y)dy$

c) $(3y - 2x)dx + (2y - 3x)dy$

d) $(4x^3 - 6xy^2)dx + (4y^3 - 6xy)dy$

25. The solution of the differential equation $\frac{dy}{dx} + y\tan x = \sec x$ is

a) $y = \sin x - C\cos x$

b) $y = \sin x + C\cos x$

c) $(y - \sin x)\sin x = C$

d) None of the above

26. Which of the following is a variable separable differential equation?

a) $(x + x^2y)dy = (2x + xy^2)dx$

b) $(x + y)dx - 2ydy = 0$

c) $y^2dx + (2y - 3y)dy = 0$

d) $2ydx = (x^2 + 1)dy$

27. The general solution of the differential equation $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$ is

a) $C_1e^{x} + C_2e^{2x} + C_3xe^{2x}$

b) $C_1e^{-x} + C_2e^{2x} + C_3xe^{2x}$

c) $C_1e^{-x} + C_2xe^{x} + C_3xe^{2x}$

d) $C_1e^{-x} + C_2e^{x} + C_3xe^{4x}$

28. The general solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x \partial y}-2\frac{\partial^2 z}{\partial y^2}+3\frac{\partial z}{\partial x}-3\frac{\partial z}{\partial y} \text{ is}$$

a) $z(x, y) = f(x+y)+e^{-3x}g(2x-y)$

b) $z(x, y) = f(x-y)+e^{-3x}g(2x+y)$

c) $z(x, y) = f(x+y)+e^{3x}g(2x+y)$

d) $z(x, y) = f(x-y)+e^{3x}g(2x-y)$

29. If $f : G \to H$ be a homomorphism of groups with kernel K. If the order of G, H and K are 75, 45 and 15 respectively, then the order of the image $f(G)$ is

a) 3 b) 5

c) 15 d) 45

30. Which of the following is not an abelian group?

a) Integers $\mathbb{Z}$

b) Permutations on two symbols S_2

c) Complex numbers $\mathbb{C}$

d) Permutations on three symbols S_3

31. How many subgroups are there for the cyclic group of order ten

a) 0 b) 1

c) 2 d) 3

32. Which of the following is not an integral domain?

a) Integers $\mathbb{Z}$

b) Rational Polynomials $\mathbb{Q}[x]$

c) Reals $\mathbb{R}$

d) $Mn(\mathbb{R})$ real $n \times n$ matrices

33. Which of the following statement is not true?

a) If R is a commutative ring with unity then so is $R[x]$

b) If R is a integral domain then so is $R[x]$

c) If R is a field so is $R[x]$

d) $R[x]/\langle x \rangle \cong R$

34. Which of the following statement is not true?

a) Every finite integral domain is a field

b) Every commutative division ring is a field

c) $\mathbb{Z}_{\alpha}$ of all integers modulo n is a field if and only if n is prime

d) The field $\mathbb{Q}$ of rational numbers contains proper subfield

35. If $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ then A (adj A) equals

a) $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$

b) $\begin{pmatrix} 0 & 10 \\ 10 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 10 & 1 \\ 1 & 10 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 10 \\ 10 & 1 \end{pmatrix}$

36. Rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ is

a) 0

b) 1

c) 2

d) 3

37. Inverse of the matrix $\begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix}$ is

a) $\begin{pmatrix} -5 & -2 \\ -3 & -1 \end{pmatrix}$

b) $\begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$

c) $\begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$

d) $\begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}$

38. Which of the following statement is always true?

a) All square matrices are invertible

b) All square matrices are diagonalizable

c) All square matrices have determinant

d) All square matrices are similar

39. The system of equations $2x + y = 5,\ x - 3y = -1,\ 3x + 4y = k$ is

a) 1 b) 2

c) 5 d) 10

40. The series $2 - 2 + 2 - 2 + ...$ is

a) convergent b) divergent

c) oscillatory d) cann't say

41. The value of $\iint_A e^{-x^2-y^2} dx\, dy$ where $A = \{(x, y) \mid x^2 + y^2 \le 4\}$

a) $\pi(e^{-4} - 1)$ b) $\pi(1 - e^{-4})$

c) $e^{\pi} - 1$ d) $\pi(e^4 - 1)$

42. The value of $\int_C y^2 dx - 2x^2 dy$ where C is the parabola $y = x^2$ from (0, 0) to (2, 4) is

a) $\frac{48}{5}$ b) $\frac{-24}{5}$

c) $\frac{24}{5}$ d) $\frac{-48}{5}$

43. Which among the following value of m gives

$$\int_1^m \int_0^1 \int_{\sqrt{x}}^1 e^{y^3} dy\, dx\, dz = e - 1$$

a) 2 b) 3

c) 4 d) 5

44. The average value of $f(x,y) = x\cos(xy)$ over the rectangle $R: 0 \le x \le \pi$ and $0 \le y \le 1$ is,

a) $\frac{\pi}{2}$ b) π

c) $\frac{2}{\pi}$ d) $\frac{2}{\pi^2}$

45. Which among the following is true for the function $f(x) = x^4 - 4x^3$

a) Concave upwards always b) Concave downwards always

c) Concave upwards in $(-\infty, 0)$ d) Concave downwards in $(2, \infty)$

46. The Laplace transform of the function te^t is

a) $\frac{1}{s-1}$ b) $\frac{1}{(s-1)^2}$

c) $\frac{1}{s+1}$ d) $\frac{1}{(s+1)^2}$

47. Inverse Laplace transform of $\frac{1}{s^2+6s+13}$ is

a) $\frac{1}{2}e^{-3t}\cos 2t$ b) $\frac{1}{2}e^{-3t}\sin 2t$

c) $\frac{1}{2}e^{3t}\cos 2t$ d) $\frac{1}{2}e^{3t}\sin 2t$

48. The sequence $\{x_n\}$ defined by $x_1 = \sqrt{7}$, $x_{n+1} = \sqrt{7 + x_n}$, for $n \ge 1$, converges to

a) $\sqrt{7}$ b) 0

c) $\frac{1-\sqrt{29}}{2}$ d) $\frac{1+\sqrt{29}}{2}$

49. The value of the series $\sum_{n=1}^{\infty} \frac{1}{2n(2n-1)}$ is

a) 1 b) $1 - \ln(2)$

c) $\ln(2) - 1$ d) $\ln(2)$

50. The function $f(x) = |x|$ is

a) Differentiable at origin

b) Continuous at origin

c) Nowhere differentiable

d) Nowhere continuous

51. Which of the following is not true for a real sequence?

a) A convergent sequence is bounded

b) A monotonic bounded sequence is convergent

c) A bounded sequence is convergent

d) A sequence is convergent if and only if it is a cauchy sequence

52. $\lim_{x \to \infty} \frac{\sin(1/x)}{1/x}$ is

a) 1

b) 0

c) –1

d) Does not exist

53. The function $f(z) = |z|$ is analytic

a) everywhere

b) nowhere

c) only at $z = 0$

d) every where except at $z = 0$

54. Consider the following power series in the complex variable z

$$f(z) = \sum_{n=1}^{\infty} n \log n \, z^n, \; g(z) = \sum_{n=1}^{\infty} \frac{e^{n^2}}{n} z^n$$

If r, R are the radii of convergence of f and g respectively, then

a) $r = 0, \; R = 1$

b) $r = 1, \; R = 0$

c) $r = 1, \; R = \infty$

d) $r = \infty, \; R = 1$

55. Let $f(z) = \frac{1}{e^z - 1}$ for all $z \in \mathbb{C}$ such that $e^z \neq 1$. Then,

a) f is a meromorphic function with simple poles at $z = i2n\pi$, $n \in \mathbb{Z}$

b) f does not have poles

c) f has removable singularities

d) f has infinitely many poles on the real axis

56. Suppose f and g are entire functions and $g(z) \neq 0$ for all $z \in \mathbb{C}$. If $|f(z)| \leq |g(z)|$, then

a) $f(z) \neq 0$ for all $z \in \mathbb{C}$

b) f is a constant function

c) $f(0) = 0$

d) $f(z) = kg(z)$ for some $k \in \mathbb{C}$

57. The value of $\int_C \frac{dz}{z+2}$ where C is $|z| = 1$ is

a) $\frac{\pi}{2}$

b) 1

c) $2\pi i$

d) 0

58. Let C be the circle of radius 2 with center at the origin in the complex plane, oriented in the anti-clock wise direction. Then the integral $\oint_C \frac{dz}{(z-1)^2}$ is equal to

a) $\frac{1}{2\pi i}$

b) $2\pi i$

c) 1

d) 0

59. Which of the following cannot be a variable in C programming?

a) Volatile

b) True

c) Friend

d) Export

60. How many number of pointer $(*)$ does C have against pointer variable declaration?

a) 7

b) 127

c) 225

d) No limit

ANSWER SHEET — PART – A

1	A	B	C	D	E	21	A	B	C	D	E	41	A	B	C	D	E
2	A	B	C	D	E	22	A	B	C	D	E	42	A	B	C	D	E
3	A	B	C	D	E	23	A	B	C	D	E	43	A	B	C	D	E
4	A	B	C	D	E	24	A	B	C	D	E	44	A	B	C	D	E
5	A	B	C	D	E	25	A	B	C	D	E	45	A	B	C	D	E
6	A	B	C	D	E	26	A	B	C	D	E	46	A	B	C	D	E
7	A	B	C	D	E	27	A	B	C	D	E	47	A	B	C	D	E
8	A	B	C	D	E	28	A	B	C	D	E	48	A	B	C	D	E
9	A	B	C	D	E	29	A	B	C	D	E	49	A	B	C	D	E
10	A	B	C	D	E	30	A	B	C	D	E	50	A	B	C	D	E
11	A	B	C	D	E	31	A	B	C	D	E	51	A	B	C	D	E
12	A	B	C	D	E	32	A	B	C	D	E	52	A	B	C	D	E
13	A	B	C	D	E	33	A	B	C	D	E	53	A	B	C	D	E
14	A	B	C	D	E	34	A	B	C	D	E	54	A	B	C	D	E
15	A	B	C	D	E	35	A	B	C	D	E	55	A	B	C	D	E
16	A	B	C	D	E	36	A	B	C	D	E	56	A	B	C	D	E
17	A	B	C	D	E	37	A	B	C	D	E	57	A	B	C	D	E
18	A	B	C	D	E	38	A	B	C	D	E	58	A	B	C	D	E
19	A	B	C	D	E	39	A	B	C	D	E	59	A	B	C	D	E
20	A	B	C	D	E	40	A	B	C	D	E	60	A	B	C	D	E

MATHEMATICS/MATHEMATICS WITH FINANCE AND COMPUTATION

PART – B

(Descriptive Type)

Answer **any eight** questions. **($8 \times 5 = 40$ Marks)**

1. Evaluate $\iint_D xy\,dA$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

2. Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

3. Prove that differentiable functions are continuous. What about the converse?

4. A hot air baloon rising straight up from a level field is tracked by a range finder 500 ft from the lift off point. At the moment the range finder's elevation angle is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 rad min. How fast is the ballon rising at that moment?

5. Prove or Disprove that if $\sum_{n=1}^{\infty} a_n$, ($a_n \geq 0$ for all $n \in \mathbb{N}$) is convergent then $\sum_{n=1}^{\infty} \frac{a_n}{1 + a_n}$ is convergent.

6. Check whether the statement "Every bounded sequence is Cauchy" true or false and justify your answer.

7. Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function such that for every $z \in \mathbb{C}$ there is some integer $n \geq 0$, satisfying $f^n(z) = 0$. Prove that f is a polynomial.

8. Prove or disprove that $\left| \dfrac{a-b}{a-\bar{a}b} \right| < 1$ where a and b are complex numbers with $|a| < 1$ and $|b| < 1$.

9. If H and K are normal subgroups of a group G such that $H \cap K = \{e\}$. Show that $hk = kh$ for all $h \in H$ and $k \in K$.

10. Let R be a Euclidean domain. Show that every ideal of R is a principal ideal.

11. Solve the differential equation $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 4y = 0$.

12. Evaluate det A where $A = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}$.