| Code No. | L – 4037 |
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**Entrance Examination for Admission to the P.G. Courses in the Teaching Departments, 2021**

**CSS**

**MATHEMATICS/MATHEMATICS WITH FINANCE AND COMPUTATION**

**General Instructions**

1. The Question Paper is having two Parts — Part 'A' Objective type (60%) & Part 'B' Descriptive type (40%).
2. Objective type questions which carry 1 mark each are to be (✓) 'tick marked' in the response sheets against the appropriate answers provided.
3. 8 questions are to be answered out of 12 questions carrying 5 marks each in Part 'B'.
4. **Negative marking** : 0.25 marks will be deducted for each wrong answer in Part 'A'.

**Time : 2 Hours** **Max. Marks : 100**

| To be filled in by the Candidate | | | | | | | | | |
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| Register Number | in Figures | | | | | | | | |
| | in words | | | | | | | | |

PART – A

(Objective Type)

Choose appropriate answer from the options in the questions. **One** mark **each**.

**($60 \times 1 = 60$ marks)**

1. The domain of convergence for $1 + x + x^2 + \ldots$ is

a) $(-1, +1)$ b) $[-1, +1]$

c) $(-2, 2)$ d) None of these

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2. The derivative of the function $f(x) = x^{2m-1}$ is

a) Even function b) Odd function

c) Constant function d) None of these

3. The radius of Convergence of the series $1 - x^2 + x^4 - x^6 + \cdots$ is

a) 1 b) Zero

c) 2 d) None of these

4. Series $\sum a_n$ convergence absolutely if

a) $\sum |a_n|$ converges

b) $\sum a_n$ converges

c) $\sum |a_n|$ diverges

d) None of these

5. The value of $\arg(z) + \arg(\bar{z}), (z \neq 0)$ is

a) 0

b) $\pi$

c) $\pi/2$

d) $\pi/4$

6. The empty set of a metric space $(X, \rho)$ is

a) Neither open nor closed

b) Open set and closed set both

c) Does not exist

d) None of these

7. If $z = a$ is an isolated singularity of *f*, then a is the pole of *f*, if

a) $\lim_{z \to a} |f(z)| = 0$

b) $\lim_{z \to a} |f(z)| = a$

c) $\lim_{z \to a} |f(z)| = \infty$

d) None of these

8. What will be the value of $\lim_{n \to \infty} \dfrac{2n^2 + 3}{3n^2 + 5n} =$

a) 2/3

b) 3/2

c) 3/5

d) 5/8

9. The name of the conic represented by the equation $2xy + 4x - 6y + 17 = 0$ is

a) an ellipse
b) a circle
c) a parabola
d) a hyperbola

10. The vertex of the parabola $x^2 + 2y = 8x - 7$ is

a) $\left(4, \frac{7}{2}\right)$
b) $\left(4, \frac{9}{2}\right)$
c) $\left(\frac{9}{2}, 4\right)$
d) $(1, 0)$

11. The equation $\frac{x^2}{10 - a} + \frac{y^2}{4 - a} = 1$ represents an ellipse, if

a) $a < 4$
b) $a > 4$
c) $4 < a < 10$
d) $a > 10$

12. The equation of the hyperbola with vertices (3,0),(-3,0) and semi latusrectum 4 is given by

a) $4x^2 - 3y^2 + 36 = 0$
b) $4x^2 - 3y^2 + 12 = 0$
c) $4x^2 - 3y^2 - 36 = 0$
d) None of these

13. The lines $3x + 4y + 6 = 0$, $\sqrt{2}x + \sqrt{3}y + 2\sqrt{2} = 0$ and $4x + 7y + 8 = 0$ are

a) sides of a triangle
b) parallel
c) concurrent
d) none of these

14. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$, is

a) 0
b) 1
c) 3
d) 4

15. Consider the following statements

    1. A differential function is continuous
    2. A continuous function is differentiable.
    3. A Continuous function on a closed interval [a,b] of finite length is uniformly continuous on [a,b].

    Which of these statements are correct?

    a) 1,2 and 3 b) 2 and 3 c) 1 and 3 d) 1 and 2

16. The function $f(x) = |x|$ at $x = 0$ is

    a) Continuous and differentiable
    b) Continuous but not differentiable
    c) Not continuous but differentiable
    d) Neither continuous nor differentiable

17. If $g$ is the inverse of $f$ and $f'(x) = \dfrac{1}{1+x^3}$, then $g'(x)$ is equal to

    a) $\dfrac{-1}{2(1+x^2)}$ b) $1+[g(x)]^3$

    c) $\dfrac{1}{2(1+x^2)}$ d) None of these

18. $\int \sec^n x \tan x \, dx$ is equal to

    a) $\dfrac{\sec^2 x}{2} + c$ b) $\dfrac{\sec^n x}{n} + c$

    c) $\dfrac{\tan x}{2} + c$ d) $\dfrac{(\sec^n x)\tan x}{n} + c$

19. $\int x^2 e^x dx$ is equal to

a) $x^2e^x - 2\left[e^{2x} - xe^x\right] + c$

b) $x^2e^x - 2\left[e^x - xe^x\right] + c$

c) $x^2e^x - 2\left[xe^{2x} - e^x\right] + c$

d) $x^2e^x - 2\left[xe^x - e^x\right] + c$

20. $\int (\log x)^2 dx$ is equal to

a) $x(\log x)^2 - 2[x \log x - x] + c$

b) $x(\log x)^2 - 2[\log x - x] + c$

c) $x(\log x)^2 - 2\left[\log x^2 - x\right] + c$

d) $x(\log x)^2 - 2[\log x - 2x] + c$

21. The value of $\int_0^{2\pi} [2 \sin x] \, dx$ is

a) $-\pi$　　b) $-2\pi$　　c) $-3\pi$　　d) $-4\pi$

22. The value of the integral $\int_1^3 |(x-1)(x-2)(x-3)| dx$ is

a) $\frac{1}{3}$　　b) $\frac{1}{2}$

c) $\frac{9}{4}$　　d) None of these

23. The limiting value of $(\cos x)^{1/\sin x}$ as $x \to 0$ is

a) 1　　b) e

c) 0　　d) None of these

24. Which of the following is not a valid variable name declaration?

a) float PI = 3.14 b) double PI = 3.14

c) int PI=3.14 d) #define PI 3.14

25. Which among the following is not a logical or relational operator?

a) != b) == c) || d) =

26. Which of the following is an invalid if-else statement?

a) if (if(a==1)){} b) if (func 1 (a)){}

c) if(a){} d) if (char ) a) {}

27. The code 'for (;;)' represents an infinite loop. It can be terminated by

a) break b) exit (0) c) abort () d) terminate

28. A first order differential equation $M(x, y)dx + N(x, y)dy = 0$ is exact if

a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ b) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

c) $\frac{\partial M}{\partial x} < \frac{\partial N}{\partial y}$ d) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

29. What is the order and degree of a differential equation $\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$

a) First order, second degree

b) First order, first degree

c) Second degree, first order

d) Second order, second degree

30. The integrating factor for the differential equation $2ydx + xdy = 0$ is

a) $x$ b) $y$ c) $xy$ d) $y^2$

31. The value of Wronskian $W(x, x^2, x^3)$ is

a) $2x^4$ b) $2x^2$ c) $2x^3$ d) None of these

32. The laplace Transform of $e^t(\sin t)$

a) $\frac{a}{a^2 + (s+1)^2}$ b) $\frac{a}{a^2 + (s-1)^2}$

c) $\frac{s+1}{a^2 + (s+1)^2}$ d) $\frac{s+1}{a^2 + (s-1)^2}$

33. The inverse of Laplace transform of $F(s) = (5s+1)/(s^2 - 25)$ is

a) $5\cosh 5t + 1/5 \sinh 5t$ b) $5\cos 5t + 1/5 \sin 5t$

c) $\cosh 5t + 1/5 \sinh 5t$ d) $5\cos 5t + \sin 5t$

34. The point (0,0) in the domain of $f(x, y) = \sin(xy)$ is a point of

a) saddle b) minima c) maxima d) constant

35. For a function $f(x, y)$ to have no extremum value at $(a, b)$ is

a) $rt - s^2 > 0$ b) $rt - s^2 < 0$

c) $rt - s^2 = 0$ d) none of the above

36. $f(x,y) = (\sin xy + x^3 y) / x + x^3$ find $f_{xy}$ at (0, 1)

a) 2 b) 5 c) 1 d) undefined

37. Laplace transform is basically an

a) differential transform
b) integral transform
c) algebraic transform
d) rational transform

38. The value of the determinant $\begin{vmatrix} m & n & p \\ p & m & n \\ n & p & m \end{vmatrix}$ is

a) $m^3 + n^3 + p^3$
b) $3mnp$
c) $m^3 + n^3 + p^3 - 3mnp$
d) zero

39. If unitary matrix $A$ is real, then $A$ is ————————.

a) symmetric matrix
b) skew-symmetric matrix
c) hermitian matrix
d) orthogonal matrix

40. The matrix $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$ has a eigen values ————————.

a) 8, 8 b) 8, 3 c) 8, 2 d) 0, 0

41. Which is the number that comes next in the following sequence

2, 6, 12, 20, 30, 42, 56, ……

a) 60 b) 64 c) 72 d) 70

42. Pointing a man, a woman said, “His mother is the only daughter of my mother”. How is the woman related to the man?

a) Mother
b) Daughter
c) Sister
d) Grandmother

43. The range of the sequence $\left(1+(-1)^{n}\right)$ is

a) $N$ b) $z$ c) {0, 1} d) {0, 2}

44. The following vectors (1,9,9,8), (2,0,0,8) and (2,0,0,3) are

a) Linearly dependent
b) Linearly independent
c) Constant
d) None of these

45. The rank of zero matrix is

a) zero
b) one
c) order of the matrix
d) none of these

46. Empty set is

a) Linearly dependent
b) Linearly independent
c) Does not span {0}
d) None of these

47. If $f : [a,b] \to R$ is continuous and monotone function, then

a) $f$ is Riemann integrable on $[a,b]$
b) $f$ is not Riemann integrable on $[a,b]$
c) $f$ is Riemann integrable on $R$
d) None of these

48. Let $S = [0,1]$ the maximal element of $S$ is

a) 0 b) 1 c) $\phi$ d) 2

49. How many different words can be made out of the letters, which form the word ALLAHABAD?

a) $\frac{4!}{9!2!}$ b) $\frac{2!}{4!9!}$ c) $\frac{9!}{4!2!}$ d) $\frac{4!2!}{9!}$

50. The number of elements in the set $\{m : 1 \le m \le 1000$, $m$ and 1000 are relatively prime$\}$ is

a) 100 b) 250 c) 300 d) 400

51. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors then the scalar triple product $\left[2\vec{a} - \vec{b}\ 2\vec{b} - \vec{c}\ 2\vec{c} - \vec{a}\right]$ is equal to

a) 0 b) 1

c) $-\sqrt{3}$ d) $\sqrt{3}$

52. A unit vector perpendicular to both the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is

a) $\dfrac{-\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$ b) $\dfrac{\hat{i} + \hat{j} - \hat{k}}{3}$

c) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ d) $\dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

53. The equation of the plane through the points (2,3,1) and (4,–5,3) and parallel to $x$-axis is

a) $x - z - 1 = 0$ b) $4x + y - 11 = 0$

c) $y + 4z - 7 = 0$ d) None of these

54. The coordinates of the point of intersection of the line $\dfrac{x+1}{1} = \dfrac{y+3}{3} = \dfrac{z-2}{-2}$ with the plane $3x + 4y + 5z = 25$ are

a) (5, 15, 10) b) (5, 15, –10)

c) (5, –15, 10) d) None of these

55. The plane $2x - (1+\lambda)y + 3\lambda z = 0$ passes through the intersection of the planes

a) $2x - y = 0$ and $y + 3z = 0$

b) $2x - y = 0$ and $y - 3z = 0$

c) $2x + 3z = 0$ and $y = 0$

d) None of the above

56. In a three-dimensional space, the equation $3x - 4y = 0$ represents

a) a plane containing $Z$-axis

b) a plane containing $X$-axis

c) a plane containing $y$-axis

d) None of these

57. The three planes $x + y = 0$, $y + z = 0$ and $x + z = 0$

a) meet in a unique point

b) meet in a line

c) meet taken two at a time in parallel lines

d) None of the above

58. The equation $x^2 + y^2 + z^2 - 4x + 6y - 8z + 29 = 0$ represents

a) a sphere

b) the empty set

c) a point

d) none of these

59. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

a) $30^\circ$

b) $45^\circ$

c) $60^\circ$

d) $90^\circ$

60. The center of the circle given by $\vec{r}.(\hat{i} + 2\hat{j} + 2\hat{k}) = 15$ and $\vec{r} - (\hat{j} + 2\hat{k}) = 4$ is

a) (1,2,4)

b) (3,1,4)

c) (1,3,4)

d) None of these

## ANSWER SHEET — PART – A

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | A | B | C | D | E | 21 | A | B | C | D | E | 41 | A | B | C | D | E |
| 2 | A | B | C | D | E | 22 | A | B | C | D | E | 42 | A | B | C | D | E |
| 3 | A | B | C | D | E | 23 | A | B | C | D | E | 43 | A | B | C | D | E |
| 4 | A | B | C | D | E | 24 | A | B | C | D | E | 44 | A | B | C | D | E |
| 5 | A | B | C | D | E | 25 | A | B | C | D | E | 45 | A | B | C | D | E |
| 6 | A | B | C | D | E | 26 | A | B | C | D | E | 46 | A | B | C | D | E |
| 7 | A | B | C | D | E | 27 | A | B | C | D | E | 47 | A | B | C | D | E |
| 8 | A | B | C | D | E | 28 | A | B | C | D | E | 48 | A | B | C | D | E |
| 9 | A | B | C | D | E | 29 | A | B | C | D | E | 49 | A | B | C | D | E |
| 10 | A | B | C | D | E | 30 | A | B | C | D | E | 50 | A | B | C | D | E |
| 11 | A | B | C | D | E | 31 | A | B | C | D | E | 51 | A | B | C | D | E |
| 12 | A | B | C | D | E | 32 | A | B | C | D | E | 52 | A | B | C | D | E |
| 13 | A | B | C | D | E | 33 | A | B | C | D | E | 53 | A | B | C | D | E |
| 14 | A | B | C | D | E | 34 | A | B | C | D | E | 54 | A | B | C | D | E |
| 15 | A | B | C | D | E | 35 | A | B | C | D | E | 55 | A | B | C | D | E |
| 16 | A | B | C | D | E | 36 | A | B | C | D | E | 56 | A | B | C | D | E |
| 17 | A | B | C | D | E | 37 | A | B | C | D | E | 57 | A | B | C | D | E |
| 18 | A | B | C | D | E | 38 | A | B | C | D | E | 58 | A | B | C | D | E |
| 19 | A | B | C | D | E | 39 | A | B | C | D | E | 59 | A | B | C | D | E |
| 20 | A | B | C | D | E | 40 | A | B | C | D | E | 60 | A | B | C | D | E |

## MATHEMATICS/MATHEMATICS WITH FINANCE AND COMPUTATION

PART – B

(Descriptive Type)

Answer **any eight** questions. **(8 $\times$ 5 = 40 Marks)**

1. Prove that if $(a^n) \to l$, $(b^n) \to l$ and $a_n \leq c_n \leq b_n$ for all $n$, then $(c_n) \to l$

2. Test the convergence of $\sum \frac{n^3 + a}{2^n + a}$.

3. Show that the function $f(z) = e^{x^2 - y^2}[\cos(2xy) + i \sin(2xy)]$ is analytic and find its derivative.

4. If $f(x)$ is analytic, show that

   (i) $\nabla^2 |\mathrm{Re}\, f(z)|^2 = 2|f'(z)|^2$.

   (ii) $\nabla^2 |\mathrm{Im}\, f(z)|^2 = 2|f'(z)|^2$.

   (iii) $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$.

5. Find the Laplace transformation of $f(t) = e^{at} \sin(bt)$.

6. Find all real solutions of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 3x$.

7. Find the triple vectors of $a \times (b \times c)$ and $(a \times b) \times c$. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + \hat{k}$.

8. Solve the equation $x + y + z = d$, $ax + by + cz = d^2$, $a^2x + b^2y + c^2z = d^3$.

9. Show the matrix $\begin{bmatrix} 2 & 1 & 4 \\ 8 & -1 & 3 \\ 3 & -5 & 0 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

10. Write a c- program that reads test scores, into the array score, adds them and prints their average.

11. Solve $\dfrac{dy}{dx} = \dfrac{x^2 + 2}{y}$.

12. Write a program to calculate the average of set of $N$ numbers.