Code No.	J – 2273

Entrance Examination for Admission to the P.G. Courses in the Teaching Departments, 2020

CSS

DATA SCIENCE

General Instructions

1. The Question Paper is having two Parts — Part 'A' Objective type (60%) & Part 'B' Descriptive type (40%).
2. Objective type questions which carry 1 mark each are to be (✓) 'tick marked' in the response sheets against the appropriate answers provided.
3. 8 questions are to be answered out of 12 questions carrying 5 marks each in Part 'B'.
4. **Negative marking** : 0.25 marks will be deducted for each wrong answer in Part 'A'.

Time : 2 Hours **Max. Marks : 100**

To be filled in by the Candidate									
Register Number	in Figures								
	in words								

PART – A

(Objective Type)

Choose appropriate answer from the options in the questions. **One** mark **each**.

(60 × 1 = 60 marks)

1. Two finite sets have *m* and *n* elements. The number of subsets of the first set is 122 more than that of the second set. The values of *m* and *n* are, respectively

   a) 4, 7 b) 7, 4

   c) 4, 4 d) 7, 7

DO NOT WRITE HERE

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2. If $A = \{x : x \text{ is an odd natural number}\}$ and $B = \{x : x \text{ is a prime number}\}$, then $A \cap B$ is
   a) the set odd natural numbers
   b) the set prime numbers
   c) the set of odd prime numbers
   d) none of these

3. In a class of 60 students, 25 students play Cricket and 20 students play Tennis, and 10 students play both the games. Then the number students which play neither is
   a) 0
   b) 25
   c) 35
   d) 45

4. The range of the function $f(x) = 1 + 3\cos 2x$ is

a) [2, 3]
b) [2, 4]
c) [–2, 4]
d) none of these

5. If $[x]^2 - 5[x] + 6 = 0$, where [ ] denotes the greatest integer function, then

a) $x \in [3, 4]$
b) $x \in [2, 3]$
c) $x \in (2, 3]$
d) $x \in [2, 4]$

6. Let $A = \{0, 1, 2, 3\}$ and define a relation $\mathcal{R}$ on A as follows :

$\mathcal{R}$ = {(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)}

Then $\mathcal{R}$ is

a) reflexive and symmetric
b) symmetric and transitive
c) reflexive and transitive
d) an equivalence relation

7. Let $\mathbb{R}$ be the set of real numbers and $f : \mathbb{R} \to \mathbb{R}$ be the functions defined by $f(x) = 4x + 5$, then f is

a) one one but not onto
b) onto but not one one
c) bijective
d) none of these

8. Set *A* has 3 elements and *B* has 4 elements. Then, the number of injective mappings that can be defined from *A* to *B* is

a) 144
b) 12
c) 24
d) 64

9. If $y^x = e^{y-x}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{(1+\log y)}{y \log y}$
b) $\frac{(1+\log y)^2}{\log y}$
c) $\frac{(1+\log y)^2}{(\log y)} C$
d) $\frac{(1+\log y)^2}{y \log y}$

10. If $\sin y = x \sin(a + y)$, then $\frac{dy}{dx}$ is equal to

a) $\frac{(\sin(a + y))}{\sin a}$

b) $\frac{\sin^2 (a + y)}{\sin a}$

c) $\frac{2(\sin(a + y))}{\sin a}$

d) $\frac{(\sin(a + y))}{\sin y}$

11. The function $f(x) = x^{1/x}$, $x > 1$ is

a) increasing in $(1, \infty)$

b) decreasing in $(1, \infty)$

c) increasing in $(1, e)$ and decreasing in (e, ∞)

d) decreasing in $(1, e)$ and increasing in (e, ∞)

12. The sum of two numbers is 3, the maximum value of the product of the first and the square of the second is

a) 4

b) 1

c) 3

d) 0

13. $\int \frac{\sin x}{\sin(x - a)} dx$ is equal to

a) $x + \sin a \cos a \log|\sin(x - a)| + c$

b) $\cos ax + \sin a \log|\sin(x - a)| + c$

c) $\sin ax + \sin a \log|\sin(x - a)| + c$

d) None of the above

14. $\int \frac{\operatorname{cosec}^2 x - 2020}{\cos^{2020}} dx$ is equal to

a) $\frac{\cot x}{(\cos x)^{2020}} + c$

b) $\frac{\tan x}{(\cos x)^{2020}} + c$

c) $\frac{-\tan x}{(\cos x)^{2020}} + c$

d) $\frac{-\cot x}{(\cos x)^{2020}} + c$

15. A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes at least 1 lady, at least 1 old man and at most 2 young men. Then the total number of ways in which this committee can be formed is

a) 40 b) 41

c) 16 d) 32

16. Mean of 5 observations is 7. If four of these observations are 6, 7, 8, 10 and one is missing then the variance of all the five observations is

a) 4 b) 6

c) 8 d) 2

17. Given sum of the first n terms of a an AP is $2n + 3n^2$. Another AP is formed with the same first term and double of the common difference, the sum of n terms of the new AP is

a) $n + 4n^2$ b) $6n^2 - n$

c) $n^2 + 4n$ d) $3n + 2n^2$

18. For integers m and n, both greater than one, consider the following three statements

$P : m|n$ (m divides n)

$Q : m|n^2$ (m divides n^2)

$R : m$ is a prime

then

a) $Q \wedge R \rightarrow P$ b) $P \wedge Q \rightarrow R$

c) $Q \rightarrow R$ d) $Q \rightarrow P$

19. Let

$$S = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} : a_{ij} \in \{0, 1, 2\}, a_{11} = a_{22} \right\}$$

Then the number matrices in the set is

a) 20 b) 27

c) 24 d) 10

20. 5 digit numbers are to be formed using 2, 3, 5, 7 without repeating the digits. If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000, the $p : q$ is

a) 6 : 5 b) 3 : 2

c) 4 : 3 d) 5 : 3

21. If $P = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α equals

a) 4 b) 0

c) 11 d) 5

22. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$ and $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ denotes the complement of the event A. Then the events A and B are

a) independent but not equally likely

b) independent and equally likely

c) mutually exclusive and independent

d) equally likely but not independent

23. If g is the inverse of the function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to

a) $1+x^5$ b) $5x^4$

c) $\frac{1}{1+g(x)^5}$ d) $1+g(x)^5$

24. Three positive numbers form an increasing GP. If the middle term of this GP is doubled, then new numbers are in AP. Then the common ratio of the GP is

a) $\sqrt{2}+\sqrt{3}$ b) $3+\sqrt{2}$

c) $2-\sqrt{3}$ d) $2+\sqrt{3}$

25. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is

a) 7 b) 5

c) 10 d) 8

26. For what values of x, the matrix $\begin{pmatrix} 5-x & x+1 \\ 2 & 4 \end{pmatrix}$ is singular?

a) 2
b) 3
c) 4
d) 5

27. The value of the determinant $\begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$ is

a) 0
b) $a+b+c$
c) –1
d) None of these

28. The range of the function $f(x) = \frac{|x-1|}{x-1}$, $x \in \mathbb{R} \setminus \{1\}$ is

a) {1}
b) {1, 0}
c) {0}
d) {–1, 1}

29. Let $A = \{x \in \mathbb{Z} : 0 \le x \le 12\}$. Define an equivalence relation on $\mathcal{R}$ on A by

$\mathcal{R} = \{(a, b) : a, b \in \mathbb{Z}, |a-b| \text{ is divisible by } 4\}$

Then the set of all elements related to 1 is

a) {1, 5, 9}
b) {1, 6, 10}
c) {1, 7, 11}
d) {1, 8, 12}

30. The maximum value of the function $\sin x + \cos x$ is

a) 2
b) 1
c) $\sqrt{2}$
d) None of these

31. The inverse of the function $f(x) = \frac{3x-4}{5}$ is given by

a) $\frac{5x+4}{3}$
b) $\frac{5x-4}{3}$
c) $\frac{4x+3}{5}$
d) None of these

32. Which of the following functions are even functions?

a) e^x b) x^3

c) $x^{2020} + x^{2018}$ d) $\sin x$

33. Which of the following functions are odd functions?

a) e^x b) $\sin x$

c) $\cos x$ d) $\sin x + \cos x$

34. The value of the integral $\int_{-2}^{2} \sin^{101} x \, dx$ is

a) 0 b) π

c) $\pi/2$ d) None of these

35. The value of the integral $\int_{-1}^{1} x|x|dx$ is

a) $\frac{1}{2}$ b) $\frac{1}{4}$

c) 0 d) 1

36. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events "A coming in time" and "B coming in time" are independent. Then the probability that only one of them coming to the school in time is

a) $\frac{8}{49}$ b) $\frac{15}{49}$

c) $\frac{26}{49}$ d) None of these

37. If $\mathcal{R} = \{(x, y) : x + 2y = 8\}$ is a relation on the set of natural numbers $\mathbb{N}$. The range of $\mathcal{R}$ is

a) $\{2, 4, 6\}$
b) $\{1, 2, 3\}$
c) $\mathbb{N}$
d) $\{1, 2, 3, 4, 5, 6\}$

38. A binary operation $* : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is defined as

$a * b = 2a + b$

The value of $(2*3)*4$ is

a) 18
b) 9
c) 24
d) None of these

39. If the arithmetic mean of a and b is $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$, then n is

a) 1
b) –1
c) 0
d) None of these

40. Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. Assume that there equal number of males and females. The probability that this person being male is

a) $\frac{10}{21}$
b) $\frac{15}{21}$
c) $\frac{20}{21}$
d) None of the above

41. The solution of the equation $(x + 1) + (x + 4) + (x + 7) + \quad + (x + 28) = 155$ is

a) –1
b) 0
c) 2
d) 1

42. If a polygon has 44 diagonals, then the number of its sides is

a) 11
b) 7
c) 8
d) None of these

43. Let $A = \frac{1^{15}+2^{15}+3^{15}+4^{15}}{4}$, $B = \frac{1^{15}+3^{15}}{2}$ and $C = \frac{2^{15}+4^{15}}{2}$. Then which of the following is true.

a) $B<A<C$

b) $A<B<C$

c) $B<C<A$

d) $C<B<A$

44. Let $f(x) = x^2 + 2x + 3$ be a polynomial in $\mathbb{Z}_{15}$. Then $f(2)$ in $\mathbb{Z}_5$ is

a) 11

b) 2

c) 0

d) 1

45. The negation of the statement $p \wedge q$ is

a) $\neg p \vee q$

b) $\neg p \vee \neg q$

c) $p \vee \neg q$

d) None of these

46. The statement $\neg(p \rightarrow q)$ is equivalent to

a) $p \vee q$

b) $\neg p \wedge \neg q$

c) $p \wedge \neg q$

d) None of these

47. Which of the following propositions is tautology?

a) $(p \vee q) \rightarrow q$

b) $p \wedge (q \rightarrow p)$

c) $p \wedge (p \rightarrow q)$

d) Both (b) and (c)

48. 7 flags of different colors, how many different signals can be generated if a signal requires the use of two flags, one below the other?

a) 13

b) 49

c) 42

d) None of the above

49. If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|$, then the value of k is

a) 3

b) 9

c) 27

d) None of these

50. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. If the side is 10 cm long, then the rate at which its area increases, is

a) $10\sqrt{3}$ cm^2/sec

b) $10\sqrt{2}$ cm^2/sec

c) $\sqrt{3}$ cm^2/sec

d) None of these

51. The range of the function $f(x) = [x]$, where $[x]$ denotes the greatest integer function is

a) the set of natural numbers

b) the set of integers

c) the set of rational numbers

d) None of these

52. Suppose A is a set containing one element. How many elements are in $\wp(\wp(A))$? $\wp(A)$ denotes the power set of A]

a) 1

b) 2

c) 3

d) 4

53. For $n \in \mathbb{Z}$, let $A_n = \{a \in \mathbb{Z} : a \leq n\}$. Then $_{i=0}^{4} A_i$ is

a) A_0

b) A_1

c) A_2

d) A_3

54. Define the function $h : \mathbb{Z}_8 \to \mathbb{Z}_8$ by $h(x) = x^3$. Then h is

a) injective

b) surjective

c) invertible

d) not invertible

55. If A is a skew symmetric matrix of order 1001, then the value of the determinant of A is

a) 0

b) 1

c) 1001

d) None of these

56. What will be the output of the following code?

```
#include<iostream>
using namespace std;
int main
{
int n;
for(n=5; n>0; n- -)
{
cout<<n;
if(n==3)
break;
}
return 0;
}
```

a) 543 b) 54

c) 5432 d) 53

57. What will be the output of the following code?

```
#include<iostream>
using namespace std;
main( )
{
int r, x=2;
float y=5;
r=y%x;
cout<<r;
}
```

a) 1 b) 0

c) compile error d) 2

58. What will be the output of the following code?

```
int main( )
{
int a=1;
switch(a)
{
case 1: cout<<“One”;
case 2: cout<<“Two”;
case 3: cout<<“Three”;
default : cout<<“Default”;
}
return 0;
}
```

a) One
b) Compilation error
c) OneTwoThree
d) Default

59. The period of the function $f(x) = |\sin^{2020} x| + |\cos^{2020} x|$ is

a) 0
b) π
c) $\pi/4$
d) None of these

60. The LCM of the numbers 2π, π and $2\pi/3$ is

a) π
b) $\pi/2$
c) 3π
d) None of these

ANSWER SHEET — PART – A

1	A	B	C	D	E	21	A	B	C	D	E	41	A	B	C	D	E
2	A	B	C	D	E	22	A	B	C	D	E	42	A	B	C	D	E
3	A	B	C	D	E	23	A	B	C	D	E	43	A	B	C	D	E
4	A	B	C	D	E	24	A	B	C	D	E	44	A	B	C	D	E
5	A	B	C	D	E	25	A	B	C	D	E	45	A	B	C	D	E
6	A	B	C	D	E	26	A	B	C	D	E	46	A	B	C	D	E
7	A	B	C	D	E	27	A	B	C	D	E	47	A	B	C	D	E
8	A	B	C	D	E	28	A	B	C	D	E	48	A	B	C	D	E
9	A	B	C	D	E	29	A	B	C	D	E	49	A	B	C	D	E
10	A	B	C	D	E	30	A	B	C	D	E	50	A	B	C	D	E
11	A	B	C	D	E	31	A	B	C	D	E	51	A	B	C	D	E
12	A	B	C	D	E	32	A	B	C	D	E	52	A	B	C	D	E
13	A	B	C	D	E	33	A	B	C	D	E	53	A	B	C	D	E
14	A	B	C	D	E	34	A	B	C	D	E	54	A	B	C	D	E
15	A	B	C	D	E	35	A	B	C	D	E	55	A	B	C	D	E
16	A	B	C	D	E	36	A	B	C	D	E	56	A	B	C	D	E
17	A	B	C	D	E	37	A	B	C	D	E	57	A	B	C	D	E
18	A	B	C	D	E	38	A	B	C	D	E	58	A	B	C	D	E
19	A	B	C	D	E	39	A	B	C	D	E	59	A	B	C	D	E
20	A	B	C	D	E	40	A	B	C	D	E	60	A	B	C	D	E

DATA SCIENCE

PART – B

(Descriptive Type)

Answer **any eight** questions. **($8 \times 5 = 40$ Marks)**

1. Define a relation ~ on the closed interval [0, 1] by

   $x \sim y$ iff $x - y$ is a rational number.

   Prove that the relation ~ is an equivalence relation on [0, 1].

2. If $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$ and $f(1) = 7$, then find the value of $\sum_{r=1}^{n} f(r)$.

3. The sum of three numbers in AP is 24 and their product is 440. Find the numbers.

4. The ratio of HM and GM of two positive numbers are in the ratio 12 : 13. Find the ratio of the numbers.

5. Consider a prepositional language where

   (i) p means "Pooja is happy"

   (ii) q means "Pooja paints a picture"

   (iii) r means "Praveen is happy"

   (a) "if Pooja is happy and paints a picture then Praveen is not happy"

   (b) "if Pooja is happy then she paints a picture"

   (c) "Pooja is happy only if she paints a picture"

6. Use truth tables method to determine whether $(p \to q) \vee (p \to \neg q)$ is valid.

7. If the matrix M_r is given by $\begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, $r = 1, 2,$

then find the value of $\det(M_1) + \det(M_2) + \cdots + \det(M_{2020})$

8. Prove that $\lim_{x \to 0} \frac{e^{-1/x^2}}{x} = 0$.

9. Bag I contains 4 white and 6 black balls while another Bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags and it is found to be black. Find the probability that it was drawn from Bag I.

10. If $\log_2 3 \log_3 4 \log_4 5 \cdots \log_n (n+1) = 10$. Find n.

11. Write an algorithm to find the largest of 3 numbers.

12. Prove that the polynomial $x^4 + 3x^3 + 2x + 4$ can be factored into linear factors in $\mathbb{Z}_5[x]$.