Code No.	J – 2277

Entrance Examination for Admission to the P.G. Courses in the Teaching Departments, 2020

CSS

MATHEMATICS/MATHEMATICS WITH FINANCE AND COMPUTATION

General Instructions

1. The Question Paper is having two Parts — Part ‘A’ Objective type (60%) & Part ‘B’ Descriptive type (40%).
2. Objective type questions which carry 1 mark each are to be (✓) ‘tick marked’ in the response sheets against the appropriate answers provided.
3. 8 questions are to be answered out of 12 questions carrying 5 marks each in Part ‘B’.
4. **Negative marking** : 0.25 marks will be deducted for each wrong answer in Part ‘A’.

Time : 2 Hours **Max. Marks : 100**

To be filled in by the Candidate									
Register Number	in Figures								
	in words								

PART – A

(Objective Type)

Choose appropriate answer from the options in the questions. **One** mark **each**.

(60 × 1 = 60 marks)

1. If $x^2 + 2xy = y^2$, then $\frac{dx}{dy}$ is

a) $\frac{x+y}{y-x}$

b) $2x+2y$

c) $\frac{x+1}{y}$

d) $-x$

DO NOT WRITE HERE

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2. If $f(x) = g(u)$ and $u = u(x)$, then the derivative is

   a) $f'(x) = g'(u)$ b) $f'(x) = g'(u) \cdot u'(x)$

   c) $f'(x) = u'(x)$ d) None of these

3. If $f(x)$ and $g(x)$ are differentiable functions such that $f'(x) = 3x$ and $g'(x) = 2x^2$, then the limit $\lim \frac{[f(x) + g(x)) - (f(1) + g(1))]}{(x-1)}$ as x approaches 1 is equal to

   a) 5 b) 0

   c) 20 d) None of these

4. Which of the following is the indefinite integral of x^2+7?

a) $\int (x^2+7)\,dx = 2x + c$

b) $\int (x^2+7)\,dx = x^3 + 7x$

c) $\int (x^2+7)\,dx = \frac{1}{2}x^3 + 7x$

d) $\int (x^2+7)\,dx = \frac{1}{3}x^3 + 7x + c$

5. The value of $I = \int e^x \cot(e^x)\,dx$ is

a) $\text{In}\left|\cos(e^x)\right| + C$

b) $\text{In}\left|\cos x\right| + C$

c) $\text{In}\left|\sin(e^x)\right| + C$

d) $\text{In}\left|\sin x\right| + C$

6. Which of the following expressions may be evaluated using the Fundamental Theorem of Calculus?

a) $\int (15x - 3x^2 - 4)\,dx$

b) $\int (15x - 3x^2 - 1)\,dx$

c) $\int (-11e^x + e - xe^x - 1)\,dx$

d) None of these

7. The area of the region bounded by $y = x^2 - 5x + 4$ and x-axis is

a) $\frac{7}{2}$

b) $\frac{5}{2}$

c) $\frac{2}{9}$

d) $\frac{9}{2}$

8. The area of inside the cardioid $r = 1 + \cos\theta$ is

a) $\frac{3\pi}{2}$

b) $\frac{\pi}{2}$

c) $\frac{2\pi}{3}$

d) $\frac{\pi}{3}$

9. The value of center and vertices of the hyperbola

$11x^2 - 25y^2 + 22x + 250y - 889 = 0$ is

a) center : (1, –5), vertices : (1, –10), (1, 0)

b) center : (–1, –5), vertices : (–1, 0), (–1, 10)

c) center : (–1, 5), vertices : (–6, 5), (4, 5)

d) center : (1, –5), vertices : (– 4, –5), (6, –5)

10. The polar equation of the curve $x^2 + y^2 = 2ax$ is

a) $r = a\cos\theta$

b) $r = 2a\cos\theta$

c) $2r = a\cos\theta$

d) $r^2 = 2a\cos\theta$

11. If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to a real number A, then $\{a_n\}_{n=1}^{\infty}$ is

a) unbounded sequence

b) bounded sequence

c) both (a) and (b)

d) none of these

12. If the plane cuts at an angle to the axis but does not cut all the generators, then what is the name of the conics form?

a) Ellipse

b) Hyperbola

c) Circle

d) Parabola

13. The value of $\int_0^1 \log x \, dx$ is

a) 0

b) 1

c) –1

d) ∞

14. If $\sum x_n$ converges absolutely then

a) $\sum x_n^2$ converges

b) $\sum x_n^2$ diverges

c) $\sum x_n^2$ neither converges nor diverges

d) None of these

15. If f is an increasing real-valued functions of a real variable then

 a) f has atmost countable number of discontinuities

 b) f does not have discontinuities

 c) f has atmost countable number of continuities

 d) None of these

16. A differentiable real valued functions on R with bounded derivative is

 a) continuous b) uniformly continuous

 c) bounded d) not continuous

17. The power series $\sum a_n Z^n$ represents an entire function if and only if

 a) $\lim\inf |a_n|^{\frac{1}{n}} = 0$

 b) $\lim\sup |a_n|^{\frac{1}{n}} = 0$

 c) $\lim\inf |a_n|^{\frac{1}{n}}$ is a finite positive number

 d) $\lim\ \inf |a_n| = 0$

18. Let z be a complex number. Then $|\operatorname{Re} z| + |\operatorname{Im} z|$ is

 a) $\leq \sqrt{2}|z|$ b) $\leq |z|$

 c) $> \sqrt{2}|z|$ d) $= \sqrt{2}|z|$

19. For what values of z does the series $\sum_{n=0}^{\infty} \left(\frac{z}{2+z}\right)^n$ converges

 a) $\text{Res}(z) < -1$ b) $\text{Res}(z) > -1$

 c) $\text{Res}(z) = -1$ d) $\text{Res}(z) = -2$

20. Let $f : \mathbb{C} \to \mathbb{C}$ be analytic such that $f(z)$ is real for all $z \in \mathbb{C}$. Then

 a) $f(z) = 0\ \forall z$ b) $f(0) = 0$

 c) $f(z) = e^z\ \forall z$ d) $f(z) = f(0)\ \forall z$

21. Let $f = u + iv$ be analytic and non-constant. Then

a) $\bar{f}$ is analytic

b) $\bar{f}$ may or may not be analytic

c) $\bar{f}$ is never analytic

d) $f + \bar{f}$ is analytic

22. The power series $\sum \frac{z^n}{n!}$ is

a) converges

b) diverges

c) converges absolutely

d) None of these

23. Let $f : \mathbb{C} \to \mathbb{C}$ be analytic and u = real part of f. Then

a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

b) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$

c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} = 0$

d) $\frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial y^3} = 0$

24. Let H be a subgroup of a group G. Then

a) H is cyclic

b) H is cyclic if G is abelian

c) H is cyclic if G is non-abelian

d) H is cyclic if G is cyclic

25. The value of the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ is

a) $2 \pm i\sqrt{6}, 3$

b) $1 \pm i\sqrt{6}, \pm\sqrt{3}$

c) $\pm i\sqrt{6}, 4$

d) $\pm 2, \pm 4$

26. An infinite cyclic group has exactly

a) three generators

b) one generator

c) two generators

d) five generators

27. Let A be non zero a square matrix of order n with $n \geq 2$, then which of the following statement is false

a) If $\det(A) = 1$ then $\operatorname{rank}(A) = 1$

b) If $\det(A) = 0$ then $\operatorname{rank}(A) \neq 0$

c) If $\det(A) \neq 0$ then $\operatorname{rank}(A) = n$

d) If $\operatorname{rank}(A) = 1$ then $\det(A) = 0$

28. A field having no proper subfield is

a) prime field
b) division ring
c) integral domain
d) none of these

29. The number of homomorphisms between the group z to z_{10} is

a) 1
b) 10
c) 5
d) 4

30. The units of the ring $Z[i]$ are

a) ± 2
b) $\pm 1, \pm i$
c) ± 1
d) $\pm i$

31. Suppose that A is a square matrix such that $B^T AB$ is diagonal for some orthogonal matrix B. Then

a) A is necessarily symmetric
b) A is skew symmetric
c) A is Hermition
d) A is skew Hermition

32. Let A be 3 $\times$ 3 upper triangular matrix whose diagonal entries are 1, 2 and –3. Then A^{-1} is

a) $\frac{7}{6}I - \frac{1}{6}A^2$
b) $\frac{2}{3}I - \frac{1}{3}A^2$
c) $3I - A^2$
d) $\frac{7}{2}A^2 - I$

33. Expansion of the matrix $\begin{vmatrix} 1 & z & -y \\ -z & 1 & x \\ y & -x & 1 \end{vmatrix}$ gives

a) $1+x+y+z$ b) $1+x^2+y^2+z^2$

c) $1+xyz$ d) $x^2+y^2+z^2$

34. If G is a finite group, then the class equation is

a) $O(G)=\sum O(Z(G))$ b) $O(G)=\sum \frac{O(G)}{O(Z)}$

c) $O(G)=\sum O(N(a))$ d) $O(G)=\sum \frac{O(G)}{O(N(a))}$

35. Let A be 3×2 matrix with $Ax=0$ if and only if $x=0$ then rank of A is

a) 1 b) 3

c) 2 d) data inadequate

36. *A* square matrix *B* is orthogonal if and only if

a) columns of *B* are mutually orthogonal unit vectors

b) columns of *B* are not mutually orthogonal identity vectors

c) columns of *B* are parallel to orthogonal unit vectors

d) none of these

37. Let P be any prime and $V=Z_p^2$, the standard two dimensional vector space over Z_p. Then the ordered bases of V is

a) $(P^2-1)(P^2+P)$ b) $(P^2-1)(P^2-P)$

c) $(P+1)(P^2-P)$ d) $(P-1)(P^2+P)$

38. The equation of the line through the origin and perpendicular to the plane $x+y-z=2$ is

a) $t\langle 1, 1, 1\rangle$ b) $t\langle 1, 0, -1\rangle$

c) $t\langle 1, 1, -1\rangle$ d) $t\langle 1, 0, 1\rangle$

39. What is the cross product $\vec{v} \times \vec{w}$ of the two vectors $\vec{v} = (0, 5, -2)$ and $\vec{w} = (3, 1, -2)$?

a) (–8, –6, 15) b) (8, –6, 15)

c) (8, 6, 15) d) (–8, –6, –15)

40. Differentiate the vector-valued function $r\,(t) = \left\langle \sin(t^2),\ 4t^2 - t \right\rangle$

a) $r'(t) = \left\langle t \cos(t^2), 8t - 1 \right\rangle$ b) $r'(t) = \left\langle 2t \cos(t), 8t - 1 \right\rangle$

c) $r'(t) = \left\langle t \cos(t^2), t - 1 \right\rangle$ d) $r'(t) = \left\langle 2t \cos(t^2), 8t - 1 \right\rangle$

41. The curve given by $\vec{r}\,(t) = t^2\,\vec{i} + 2\sin t\,\vec{j} + 2\cos\vec{k}$, then the unit tangent vector is

a) $\dfrac{2t}{\sqrt{4t^2+4}}\vec{i} + \dfrac{2\cos t}{\sqrt{4t^2+4}}\vec{j} - \dfrac{2\sin t}{\sqrt{4t^2+4}}\vec{k}$

b) $\dfrac{2t}{\sqrt{4t^2+4}}\vec{i} - \dfrac{2\cos t}{\sqrt{4t^2+4}}\vec{j} + \dfrac{2\sin t}{\sqrt{4t^2+4}}\vec{k}$

c) $\dfrac{2t}{\sqrt{4t^2+4}}\vec{i} + \dfrac{2\sin t}{\sqrt{4t^2+4}}\vec{j} - \dfrac{2\cos t}{\sqrt{4t^2+4}}\vec{k}$

d) $\dfrac{2t}{\sqrt{4t^2+4}}\vec{i} - \dfrac{2\sin t}{\sqrt{4t^2+4}}\vec{j} + \dfrac{2\cos t}{\sqrt{4t^2+4}}\vec{k}$

42. If the scalar product (dot product) of two unit vectors is zero, then they are

a) linearly dependent

b) forming an orthonormal basis

c) pointing in the same direction

d) at an angle of 180 degrees to each other

43. The dimension of the vector space of all real numbers *R* over the field of rational number is

a) 1 b) 3

c) finite d) None of the above

44. The partial differential equation of the family of surface $z=(x+y)+A(xy)$ is

a) $x\,p-y\,q=0$ b) $x\,p-y\,q=x-y$

c) $x\,p+x\,q=x+y$ d) $x\,p+y\,q=0$

45. Classify the partial differential equation $u_{xx}+u_{yy}=u_{zz}$

a) Parabolic b) Elliptic

c) Hyperbolic d) None of these

46. The gradient vector of $f(x, y)=2xy+x^2+y$ at (0, –1) is

a) $\langle 2, 1\rangle$ b) $\langle 2, -1\rangle$

c) $\langle 2, \pm 1\rangle$ d) $\langle -2, 1\rangle$

47. The double integral $\iint_R x y^2\, dx\, dy$ over the region $R=\{(x, y)\,|\,1\le x\le 5,\ 0\le y\le 2\}$ is

a) –64 b) –1

c) 1 d) 64

48. The integral of $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$ is given by

a) 0 b) 1

c) 0.25 d) 4

49. The general solution of the equation $y\,dx-x\,dy=0$ is

a) $\dfrac{x}{y}=c$ b) $x+y=c$

c) $xy=c$ d) $x-y=c$

50. In linear ordinary differential equation, the dependent variable and it differential coefficient are not multiple together and occurs only in

a) First degree
b) Second degree
c) Third degree
d) Fourth degree

51. Suppose y is a function of x which of the following is $d(x^3y)dx$

a) $3x^2y + x^3\,dy\,dx$
b) $3x^2\,y$
c) $3x^2\,dy\,dx$
d) $3x^2y + x^3$

52. The initial value problem $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$, $u = x$ on $x^2 + t^2 = 1$ has

a) unique solution
b) infinitely many solution
c) two solution
d) no solution

53. Consider the ODE $y'' - y = 0$. Then

a) $y_1 = e^2t$ and $y_2 = e^{-t^2}$ are the two solutions of the differential equation
b) The Wronskian is 5
c) The solution does not form a fundamental set
d) The solution form a fundamental set

54. Inverse Laplace transform of $\frac{1}{(s-1)(s+5)}$ is

a) $\frac{1}{6}e^{-t} - \frac{1}{36}e^{t} + \frac{1}{36}e^{-5t}$
b) $\frac{1}{6}e^{t}t - \frac{1}{36}e^{t} + \frac{1}{36}e^{-5t}$
c) $\frac{1}{6}e^{-t}\,t^2 - \frac{1}{36}e^{-t} + \frac{1}{36}e^{5t}$
d) $\frac{1}{6}e^{-t}\,t - \frac{1}{36}e^{-t} + \frac{1}{36}e^{5t}$

55. Laplace transform is very similar to the

a) differential transform
b) fourier transform
c) integral transform
d) exponential transform

56. Laplace transform of function $f(t)=t$ where $t \geq 0$ is

a) s

b) $\frac{1}{s}$

c) $\frac{1}{s^2}$

d) $\frac{1}{s^3}$

57. What will happen if in a *C* program you assign a value to an array element whose subscript exceeds the size of array?

a) The element will be set to 0

b) The computer would report an error

c) The program may crash if some important data gets overwritten

d) The array size would appropriately grow

58. What would be the equivalent pointer expression for referring the array element $a[i]\ [j]\ [k]\ [l]$?

a) $((((a+i)+j)+k)+l)$

b) $^*(^*(^*(^*(a+i)+j)+k)+l)$

c) $(((a+i)+j)+k+l)$

d) $((a+i)+j+k+l)$

59. First operating system designed using C programming language

a) DOS

b) Windows

c) UNIC

d) Mac

60. During preprocessing, the code "#include<*stdio.h*>" get replaced by the contents of the file *stdio.h*

a) Yes

b) During linking the code "#include<*stdio.h*>" replaces by *stdio.h*

c) During execution the code "#include<*stdio.h*>" replaces by *stdio.h*

d) During editing the code "#include<*stdio.h*>" replaces by *stdio.h*

ANSWER SHEET — PART – A

1	A	B	C	D	E	21	A	B	C	D	E	41	A	B	C	D	E
2	A	B	C	D	E	22	A	B	C	D	E	42	A	B	C	D	E
3	A	B	C	D	E	23	A	B	C	D	E	43	A	B	C	D	E
4	A	B	C	D	E	24	A	B	C	D	E	44	A	B	C	D	E
5	A	B	C	D	E	25	A	B	C	D	E	45	A	B	C	D	E
6	A	B	C	D	E	26	A	B	C	D	E	46	A	B	C	D	E
7	A	B	C	D	E	27	A	B	C	D	E	47	A	B	C	D	E
8	A	B	C	D	E	28	A	B	C	D	E	48	A	B	C	D	E
9	A	B	C	D	E	29	A	B	C	D	E	49	A	B	C	D	E
10	A	B	C	D	E	30	A	B	C	D	E	50	A	B	C	D	E
11	A	B	C	D	E	31	A	B	C	D	E	51	A	B	C	D	E
12	A	B	C	D	E	32	A	B	C	D	E	52	A	B	C	D	E
13	A	B	C	D	E	33	A	B	C	D	E	53	A	B	C	D	E
14	A	B	C	D	E	34	A	B	C	D	E	54	A	B	C	D	E
15	A	B	C	D	E	35	A	B	C	D	E	55	A	B	C	D	E
16	A	B	C	D	E	36	A	B	C	D	E	56	A	B	C	D	E
17	A	B	C	D	E	37	A	B	C	D	E	57	A	B	C	D	E
18	A	B	C	D	E	38	A	B	C	D	E	58	A	B	C	D	E
19	A	B	C	D	E	39	A	B	C	D	E	59	A	B	C	D	E
20	A	B	C	D	E	40	A	B	C	D	E	60	A	B	C	D	E

MATHEMATICS/MATHEMATICS WITH FINANCE AND COMPUTATION

PART – B

(Descriptive Type)

Answer **any eight** questions. **(8 $\times$ 5 = 40 Marks)**

1. Calculate the derivative of the function $g(x) = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sqrt{\sin^2 t + 2}\, dt$ at $x = \frac{\pi}{6}$.

2. Determine the area that lies inside $r = 3 + 2\sin\theta$ and outside $r = 2$.

3. Prove that *e* is irrational.

4. Given the function $f(x) = \begin{cases} \sin x & \text{if } \quad x \le \frac{-\pi}{2} \\ a \sin x + b & \text{if } \quad \frac{-\pi}{2} < x < \frac{\pi}{2} \\ 2\cos x & \text{if } \quad x \ge \frac{-\pi}{2} \end{cases}$

   Determine *a* and *b* so that the function $f(x)$ is continuous for all values of *x*.

5. Show that the function $f(z) = \bar{z} = x - i\,y$ is nowhere differentiable.

6. Prove that the set of all 2 $\times$ 2 real matrices forms a ring under the usual matrix addition and multiplication.

7. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$.

8. Find the directional derivative of $f(x, y) = \dfrac{x}{x^2 + y^2}$ in the direction of $\vec{v} = \langle 3, 5 \rangle$ at the point (1, 2).

9. Evaluate $\int_0^1 \int_0^3 \int_0^5 (x + y + z)\, dx\, dy\, dz$.

10. Solve the differential equation $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = x e^x \cos x$.

11. Find $L^{-1}\left\{\dfrac{1}{(s^2 + a^2)^2}\right\}$.

12. What is meant by single dimensional array? Explain with C program.