DEPARTMENT OF STATISTICS
UNIVERSITY OF KERALA

M. PHIL PROGRAMME IN STATISTICS

Syllabus

(Under Credit and Semester System with effect from 2016 Admission)

UNIVERSITY OF KERALA
DEPARTMENT OF STATISTICS

M Phil Programme in Statistics

Aim: The M Phil Programme in Statistics aims to prove a strong awareness about recent developments in the core areas of Statistics, particularly in the fields of Distribution Theory, Order Statistics and Reliability Theory so as make the students competent to do research in these areas.

Objectives: (1) To introduce the students to advanced areas of research in Statistics, particularly in the fields of Distribution Theory, Order Statistics and Reliability Theory (2) To equip students to undertake serious research studies in applied areas of statistics, with the acquired theoretical background and research training and (3) To enable students to become efficient teachers and researchers in Mathematical Statistics.

Structure of the Programme

Semester No.	Course code	Name of the Course	Number of Credits
I	STA-711	Research Methodology	4
	STA-712	Research Methods in Statistics	4
	STA-713 (i)	Distribution Theory	4
	STA-713 (ii)	Order Statistics	4
	STA-713 (iii)	Reliability Theory	4
II	STA-721	Dissertation	**20**
		TOTAL CREDITS	**32**

SYLLABUS FOR M. Phil. IN STATISTICS

Semester: I
Course Code: STA-711
Course Title: RESEARCH METHODOLOGY
Credits: 4

Aim: This course is designed with the aim of providing a strong awareness about various aspects of research methodology so as to equip the students to do research in mathematical statistics.

Objectives: The course will consists of lectures and related activities that will help the students to develop good understanding of the methods of research process and management. This should also equip them with regard to critical thinking, better scientific communication, use of mathematical as well as statistical packages, basic tools in theoretical research especially suitable for research in statistical sciences.

Module I: **Objectives and types of research**: Motivation and objectives - Research methods vs Methodology. Types of research - Descriptive vs. Analytical, Applied vs. Fundamental, Quantitative vs. Qualitative, Conceptual vs. Empirical. Research Formulation : Defining and formulating the research problem - Selecting the problem - Necessity of defining the problem - Importance of literature review in defining a problem - Literature review web as a source - searching the web - Critical literature review - Identifying gap areas from literature review - Development of working hypothesis.

Module II: - Research design and methods : Basic Principles and need of research design — Features of good design - Important concepts relating to research design - Developing a research plan - Exploration, Description, Diagnosis, Experimentation. Determining experimental and sample designs. Computational methods using *Matlab*, *Mathcad* and *Mathematica*. Reporting and thesis writing : Structure and components of scientific reports - Types of report - Technical reports and thesis - Significance - Different steps in the preparation - Layout, structure and Language of typical reports - Illustrations and tables - Bibliography, referencing and footnotes - LaTeX typesetting, Oral presentation - Planning - Preparation - Practice - Making presentation - Use of visual aids - Importance of effective communication.

Module III: Functions of random variables and random vectors: Method of distribution function, Method of transformations, Generating functions, Distribution of statistics based on samples from a normal population.

Module IV: Distribution Theory of Order statistics: distribution of single order statistic and joint distribution of two or more order statistics, conditional distributions, Markov chain property of order statistics, order statistics from discrete distributions, moments of order

statistics and recurrence relation on the moments of order statistics, special properties of order statistics arising from uniform, exponential, normal and logistic distributions.

Module V: Basic concepts in reliability: Failure rate, mean, variance and percentile residual life, identities connecting them; Notions of ageing - IFR, IFRA, NBU, NBUE, DMRL, HNBUE, NBUC etc and their mutual implications; TTT transforms and characterization of ageing classes.

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Semester: I
Course Code: STA-712
Course Title: RESEARCH METHODS IN STATISTICS
Credits: 4

Aim: The main aim of this course is to equip the students with sound knowledge in the basic concepts of advanced research methods in Mathematical Statistics.

Objectives: The course will consists of lectures and related activities that will enable the students to achieve sound knowledge in the advanced core areas of Mathematical Statistics. This should equip them to achieve advanced research methods in Mathematical Statistics, so that they will capable for developing new theories in the area of Statistical Theory and practice.

Module I: Families of distributions: Systems of distributions due to (a) Pearson (b) Burr (c) Johnson (d) Tadikamallla& Johnson, General concepts of convex and log-concave densities with special emphasis to log-normal, inverse Gaussian, Pareto, Weibull, extreme value, logistic distributions and generalized logistic, life time distributions and extended generalized hyper-geometric recast families of distributions.

Module II: Characterizations of distributions: Characterization of normal law from different hypothesis, structural set-up, independence of linear forms, independence of linear and quadric forms, regression and solutions of certain functional equations, characterizations based on order statistics of exponential and Weibull distributions. Characterizations of lifetime distributions bases on failure rate and mean residual life.

Module III: Nonparametric methods: power of nonparametric tests, consistency of tests, Lehmann's theorem, Asymptotic Relative Efficiency (ARE), definition of ARE in Pitman's sense, Noethen's theorem, examples of evaluation of ARE of tests. Definition and examples of U-Statistics, variance of U-Statistics, one sample U-Statistics theorem. Two sample U-Statistics and examples.

Module IV: Basic elements of Bayesian Inference, Loss function, Prior distribution, Bayes Theorem, Posterior distributions, Bayes risk, Bayes principle, Bayes estimators, Minimax estimators.

Module V: Random number generation- Acceptance –Rejection Method, Metropolis-Hasting algorithm, MCMC method, Gibbs sampling, Adaptive MCMC method, Bayes estimation using MCMC method, Bayes estimation using Lindley's approximation method.

REFERENCES

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Semester: I
Course Code: STA-713(i)
Course Title: DISTRIBUTION THEORY
Credits: 4

Aim: The aim of this course is to introduce the student with the advanced knowledge in recent trends in Distributions Theory. This course equips the students to do research in the area of distribution theory.

Objectives: The course will consists of lectures and related activities that will help the students in achieving sound knowledge in recent developments in the area of Distributions Theory. This should help them competent to do research in the area so that they are capable for developing new theories in the area of Distributions Theory and its related fields.

Module I: Poisson related distributions- hyper-Poisson distribution, alternative hyper-Poisson distribution, intervened Poisson distribution, intervened generalized Hermite distribution, grouped Poisson distribution, binomial-Poisson distribution, zero-inflated Poisson distribution, alpha generalized hyper-Poisson distribution.

Module II: Stopped sum distributions- Multiple Poisson, Poisson - Binomial and Neyman type A distribution; Distribution of order k - Poisson, geometric, negative binomial, logarithmic, hyper-Poisson, alternative hyper-Poisson and zero-inflated logarithmic distributions of order k; families of discrete distributions of Order k; Lagrangian distributions - Lagrangian Poisson, negative binomial and logarithmic series distributions.

Module III: Bivariate Discrete Distributions- bivariate Poisson, binomial, negative binomial, logarithmic, hyper-Poisson and distributions - properties and applications; Families of bivariate discrete distributions.

Module IV: Mixtures of normal distributions; skew normal and its generalized versions, two-piece skew normal and its generalizations; exponentiated type distributions, skew logistic and modified skew logistic distributions, Kies, exponentiated reduced Kies and inverse-Kies distributions.

Module V: Bivariate Continuous Distributions- bivariate exponential model of Gumbel, Marshall-Olkin, Freund, Moran; bivariate extreme value distributions; Multivariate beta and gamma distributions.

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- Kumar, C.S. and Riyaz, A.(2015). A zero-inflated logarithmic series distribution of order k and its applications, AStA Advances in Statistical Analysis, 99, 31 – 43.
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Semester: I

Course Code: STA-713(ii)
Course Title: ORDER STATISTICS
Credits: 4

PRE-REQUISITES, IF ANY: M. Phil level knowledge of STA712 Research Methods in Statistics, post graduate level knowledge in Order Statistics.

Aim: The aim of this course is to introduce the student with the concept of order statistics and its properties. This course equips the students to do research in the area of order statistics.

Objectives: The course will consists of lectures and related activities that will enable the students to get an understanding about the basic concept of ordered random variables such as order statistics and record values. This should equip them to do research in the area.

Module I: Basic distribution theory of order statistics: Distributions of some simple functions of order statistics, Moments of order statistics, identities, recurrence relations and approximations. Order statistics from specific continuous distributions such as normal, exponential, weibull, logistics and uniform.Asymptotic distribution of sample quantities.

Module II: Linear estimation of location and scale parameters based on order statistics. Lloyd's BLUE's, Gupta's simplified linear estimators, Blom's unbiased nearly best linear estimators and Downton's linear estimators. Quick estimators based on order statistics of location and scale parameters.

Module III: Approximate maximum likelihood estimation for type I and type II censoring. Tiku's modified maximum likelihood estimation. Linear estimation based on selected order statistics: Bennett's, Jung's and Ogava's optimal estimators. Balakrishnan's approximate maximum likelihood estimators.Illustrations of these methods for normal, exponential and lognormal distributions. U-statistics based on best linear functions of order statistics as kernels.

Module IV: Concomitants of Order Statistics, Concomitants of Order Statistics arising from Mongestern family of distributions, Applications of concomitants of order statstics in ranked set sampling, record values, record values arising from special distributions such as uniform, exponential and normal distributions, concomitants of record values and applications in parameter estimation.

Module V: Bayesian estimation of parameters based on order statistics, record values, ranked set sample and Type II progressive censored order statistics. Illustration of these methods to exponential, generalized exponential, generalized Pareto and Weibull distributions. exponential and lognormal distributions. U-statistics based on best linear functions of order statistics as kernels.

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Semester: I

Course Code: STA-713(iii)

Course Title: RELIABILITY THEORY

Credits: 4

Aim: The aim of this course is to introduce the student with the recent developments in concepts in reliability. This course equips the students to do research in the area of reliability theory.

Objectives: The course will consists of lectures and related activities that will enable the students to get an understanding about the basic concept of reliability theory. This should equip them to do research in the area of reliability theory.

Module I: Failure rate, mean residual life, mean time to failure in the univariate and bivariate cases. Notion of ageing based on failure rate and mean residual life, NBU, NBUE, HNBUE classes, their duals and interrelationships.

Module II: Study of life time models - Exponential, Weibull, Pareto, Inverse Gaussian and Gamma with reference to basic concepts and ageing characteristics.

Module III: Discrete time failure models:- Definition of basic functions and their properties; Ageing classes and their mutual implications, Reliability systems with dependents components:-Parallel and series systems, k out of n systems, ageing properties with dependent and independents components

Module IV: Censoring and Truncation - Right Censoring, Left or Interval Censoring, Truncation, Likelihood Construction for Censored and Truncated Data.

Module V: Reliability estimation using MLE - exponential, Weibull and gamma distributions based on censored and non censored samples; UMVUE estimation of reliability function; Bayesian reliability estimation of exponential and Weibull models.

REFERENCES

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Semester: II

Course Code: STA-721

Course Title: Dissertation

Credits: 20

Aim: This Course is designed with the aim of providing hands own training for developing a research thesis so as to get awareness about recent developments in a particular selected topics in the areas of Statistics, depending on the specialisation of the respective research supervisors.

Objectives: This Course will consist of discussions, seminars and related activities that will help the students to develop clear understanding of how to execute research process in the respective specialized areas. This should also equip the students to find research problems, better scientific presentation, use of advance statistical packages etc. which are suitable for research in statistical sciences.